A Preliminary Reification of Argument Theory Change

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Abstract In this article we introduce the basics for understanding the mechanisms of Argument Theory Change. In particular we reify it using Defeasible Logic Programming. In this formalism, knowledge bases are represented through defeasible logic programs. The main change operation we define over a defeasible logic program is a special kind of revision that inserts a new argument and then modifies the resulting program seeking for the argument’s warrant. Since the notion of argument refers to a set of defeasible rules, we generalize this technique in order to handle extended arguments, i.e., arguments containing also strict rules. Hence, revision using extended arguments allows us to consider program-independent arguments, which brings about new issues. A single notion of minimal change is analyzed, which refers to keep the contents of the program as much as possible. Finally, a brief discussion about the relation between our approach and the basic theory of belief revision is exposed, along with a description of other possible (more complex) minimal change principles.

Keywords: belief revision, argumentation, defeasible logic programming, non-monotonic reasoning.

1 Introduction & Motivation

This work presents an introduction to revision in argumentation systems, namely Argument Theory Change, using Defeasible Logic Programming (DeLP) [6] as base formalism. The objective of this article is to give the basics of this theory by defining a simple argument revision operator that ensures warrant of the conclusion of the (external) argument being added to a program. In that sense, this operator will be prioritized. Therefore, when we revise a program $P$ by an argument $\langle \mathcal{A}, \alpha \rangle$, the program resulting from the revision will be such that $\mathcal{A}$ is an undefeated argument and $\alpha$ is then warranted. Because of this, we named the operator Warrant-Prioritized Revision Operator.

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This line of research constitutes ongoing work, and two papers have already been accepted for publication. The first of them presents an abstract approach to argument theory change [16], which involves the definition of a new framework for abstract argumentation capable of successfully handling dynamics. The other article presents the corresponding reification to the DeLP formalism [14], considering an alternative to the minimal change principle presented here. It also presents a Prolog-like algorithm that implements the revision operator in a modular way.

The main issue underlying warrant-prioritized argument revision lies in the selection of arguments and the incisions that have to be made over them. Incisions will make these arguments “disappear”, but selections have to be done carefully, following some minimal change principle. In this work, we present one of the many minimal change principle that could be defined [14]. The corresponding selection and incision functions are defined, along with some properties they should verify. This revision operator is therefore reconsidered, by applying it to extended arguments.

The article is structured as follows: Section 2 gives an overview of the main concepts involved in the DeLP formalism, Section 3 describes the notions of the belief revision theory we inspired from to define this approach, Section 4 explains in detail the two versions of the revision operator, and Section 5 gives a final discussion on revision in argument systems and poses future lines of work.

2 DeLP Overview

Declarative Logic Programming (DeLP) combines results of Logic Programming and Defeasible Argumentation. The system is fully implemented and is available online [1]. A brief explanation is included below (see [6] for full details). A DeLP-program \( P \) is a set of facts, strict rules, and defeasible rules. Facts are ground literals representing atomic information or the negation of atomic information using the strong negation “\( \sim \)”, e.g., \( \text{chicken}(<\text{little}) \) or \( \sim \text{scared}(<\text{little}) \). Strict Rules represent non-defeasible information and are denoted \( L_0 \leftarrow L_1, \ldots, L_n \), where \( L_0 \) is a ground literal and \( \{L_i\}_{i>0} \) is a set of ground literals, e.g., \( \text{bird} \leftarrow \text{chicken} \) or \( \sim \text{innocent} \leftarrow \text{guilty} \). Defeasible Rules represent tentative information and are denoted \( L_0 \leftarrow L_1, \ldots, L_n \), where \( L_0 \) is a ground literal and \( \{L_i\}_{i>0} \) is a set of ground literals, e.g., \( \sim \text{flies} \leftarrow \text{chicken} \) or \( \text{flies} \leftarrow \text{chicken}, \text{scared} \).

When required, \( P \) is denoted \((\Pi, \Delta)\), distinguishing the subset \( \Pi \) of facts and strict rules, and the subset \( \Delta \) of defeasible rules (see Ex. 1). Strong negation is allowed in the head of rules, and hence may be used to represent contradictory knowledge. From a program \((\Pi, \Delta)\) contradictory literals could be derived. Nevertheless, the set \( \Pi \) (which is used to represent non-defeasible information) must possess certain internal coherence, i.e., no pair of contradictory literals can be derived from \( \Pi \).

A defeasible rule is used to represent tentative information that may be used if nothing could be posed against it. Observe that strict and defeasible rules are ground. However, following the usual convention [12], some examples use “schematic rules” with variables. To distinguish variables, as usual, they start with an uppercase letter.

**Example 1** Let \((\Pi_1, \Delta_1)\) be a DeLP-program:

\[
\Pi_1 = \begin{cases}
(bird(X) \leftarrow \text{chicken}(X)) \\
\text{chicken}(<\text{little}) \\
\text{chicken}(<\text{tina}) \\
\text{scared}(<\text{tina})
\end{cases}
\]

\[
\Delta_1 = \begin{cases}
\text{flies}(X) \leftarrow \sim \text{bird}(X) \\
\text{flies}(X) \leftarrow \text{chicken}(X), \sim \text{scared}(X) \\
\sim \text{flies}(X) \leftarrow \text{chicken}(X)
\end{cases}
\]

This program has three defeasible rules representing tentative information about the flying ability of birds in general, and about regular chickens and scared ones. It also has a strict rule expressing that every chicken is a bird, and three facts: ‘tina’ and ‘little’ are chickens, and ‘tina’ is scared.

From a program is possible to derive contradictory literals, e.g., from \((\Pi_1, \Delta_1)\) of Ex. 1 it is possible to derive \(\text{flies}(\text{tina})\) and \(\sim \text{flies}(\text{tina})\). For the treatment of contradictory knowledge DeLP incorporates a
defeasible argumentation formalism. This formalism allows the identification of the pieces of knowledge that are in contradiction, and a dialectical process is used for deciding which information prevails as warranted. This dialectical process (see below) involves the construction and evaluation of arguments that either support or interfere with the query under analysis. As we will show next, arguments supporting the answer for a given query will be shown in a particular way using dialectical trees. The definition of dialectical tree will be included below, but first, we will give a brief explanation of other related concepts (for the details see [6]).

Definition 1 (Argument Structure)
Let $(\Pi, \Delta)$ be a DeLP-program, $(\mathcal{A}, \alpha)$ is an argument structure for a literal $\alpha$ from $(\Pi, \Delta)$, if $\mathcal{A}$ is the minimal set of defeasible rules ($\mathcal{A} \subseteq \Delta$), such that: (1) there exists a defeasible derivation for $\alpha$ from $\Pi \cup \mathcal{A}$, and (2) the set $\Pi \cup \mathcal{A}$ is non-contradictory.

Example 2 From the DeLP-program $(\Pi_1, \Delta_1)$ the following arguments can be obtained (note that $f$ stands for flies; b, for bird; c, for chicken; s, for scared; and t, for tina):

- $\langle A_1, f(t) \rangle = \{\{f(t) \leftarrow b(t)\}, f(t)\}$
- $\langle A_2, \sim f(t) \rangle = \{\{\sim f(t) \leftarrow c(t)\}, \sim f(t)\}$
- $\langle A_3, f(t) \rangle = \{\{f(t) \leftarrow c(t), s(t)\}, f(t)\}$

A literal $L$ is warranted if there exists a non-defeated argument $\mathcal{A}$ supporting $L$. To establish if $(\mathcal{A}, \alpha)$ is a non-defeated argument, defeaters for $(\mathcal{A}, \alpha)$ are considered, i.e., counter-arguments that by some criterion are preferred to $(\mathcal{A}, \alpha)$. In DeLP, the comparison criterion is usually generalized specificity, but in the examples given in this paper we will abstract away this criterion, since in this work it introduces unnecessary complications. Since defeaters are arguments, there may exist defeaters for them, and defeaters for these defeaters, and so on. Thus, a sequence of arguments called argumentation line is constructed, where each argument defeats its predecessor. To avoid undesirable sequences, that may represent circular or fallacious argumentation lines, in DeLP an argumentation line is acceptable if it satisfies certain constraints (see [6]).

Example 3 From Ex. 2, we have that argument $\langle A_2, \sim f(t) \rangle$ defeats $\langle A_1, f(t) \rangle$, argument $\langle A_3, f(t) \rangle$ is a defeater for $\langle A_2, \sim f(t) \rangle$, and the arguments sequence $[\langle A_1, f(t) \rangle, \langle A_2, \sim f(t) \rangle, \langle A_3, f(t) \rangle]$ is an acceptable argumentation line.

Clearly, there might be more than one defeater for a particular argument. Therefore, many acceptable argumentation lines could arise from one argument, leading to a tree structure.

Definition 2 (Dialectical tree [6]) From a DeLP-program $\mathcal{P}$, a dialectical tree for $(\mathcal{A}_0, h_0)$, denoted $T_P((\mathcal{A}_0, h_0))$, is defined as follows:

1. The root of the tree is labelled with $(\mathcal{A}_0, h_0)$.

2. Let $N$ be a node of the tree labelled $(\mathcal{A}_n, h_n)$, and $\Lambda = [(\mathcal{A}_0, h_0), (\mathcal{A}_1, h_1), \ldots, (\mathcal{A}_n, h_n)]$ be the sequence of labels of the path from the root to $N$. Let $\{B_1, q_1\}, \{B_2, q_2\}, \ldots, \{B_k, q_k\}$ be the set of all the defeaters for $(\mathcal{A}_n, h_n)$. For each defeater $(\mathcal{B}_i, q_i)$ $(1 \leq i \leq k)$, such that, the argumentation line $N' = [(\mathcal{A}_0, h_0), (\mathcal{A}_1, h_1), \ldots, (\mathcal{A}_n, h_n), (\mathcal{B}_i, q_i)]$ is acceptable, then the node $N$ has a child $N_i$ labelled $(\mathcal{B}_i, q_i)$. If there is no defeater for $(\mathcal{A}_n, h_n)$ or there is no $(\mathcal{B}_i, q_i)$ such that $N'$ is acceptable, then $N$ is a leaf.

In a dialectical tree, every node (except the root) represents a defeater of its parent, and leaves correspond to non-defeated arguments. Each path from the root to a leaf corresponds to a different acceptable argumentation line. A dialectical tree provides a structure for considering all the possible acceptable argumentation lines that can be generated for deciding whether an argument is defeated. We call this tree dialectical because it represents an exhaustive dialectical analysis for the argument in its root.

Given a literal $h$ and an argument $(\mathcal{A}, \alpha)$ from a program $\mathcal{P}$, to decide whether a literal $h$ is warranted, every node in the dialectical tree $T_P((\mathcal{A}, h))$ is recursively marked as “D” (defeated) or “U” (undefeated), obtaining a marked dialectical tree as follows:
1. All leaves in $T_P(A,h)$ are marked as “U”s;
2. let $\langle B, q \rangle$ be an inner node of $T_P(A,h)$, then $\langle B, q \rangle$ will be marked as “U” iff every child of $\langle B, q \rangle$ is marked as “D”. The node $\langle B, q \rangle$ will be marked as “D” iff it has at least a child marked as “U”.

Given an argument $\langle A, h \rangle$ obtained from $P$, if the root of $T_P(A,h)$ is marked as “U”, then we will say that $T_P(A,h)$ warrants $h$ and that $h$ is warranted from $P$.

In this paper, marked dialectical trees will be depicted as a tree of triangles where edges denote the defeat relation (in Figure 1, three marked dialectical trees are shown). An argument $\langle A, h \rangle$ will be depicted as a triangle, where its upper vertex is labelled with the conclusion $h$, and the set of defeasible rules $A$ are associated with the triangle itself. Gray triangles will be undefeated arguments, whereas white triangles will depict defeated arguments. In the rest of the article we will refer to marked dialectical trees just as “D-tree”.

![D-tree for flies(tina)](image)

**Example 4** (Extends Ex. 3) Figure 1 shows the D-tree for $T_PA_1(A_1,f)$ (the leftmost tree), which has only one argumentation line. Observe that the argument $\langle A_2, \sim f \rangle$ interferes with the warrant of ‘flies(tina)’ and the argument $\langle A_1, f \rangle$ reinstates $\langle A_1, f \rangle$. The root of $T_PA_1(A_1,f)$ is marked as “U” and therefore the literal ‘flies(tina)’ is warranted.

### 3 Belief Revision Overview

A **belief base** is a knowledge state represented by a set of sentences not necessarily closed under logical consequence. A **belief set** is a set of sentences in a given language, closed under logical consequence. In general, a belief set is infinite being this the main reason of the impossibility to deal with this kind of sets in a computer. Instead, it is possible to characterize the properties that each of the change operations should satisfy on any finite representation of a knowledge state.

Classic operations in the theory change [2] are known as expansions, contractions, and revisions. An **Expansion** operation, noted with “+”, adds a new belief to the epistemic state, without guaranteeing its consistency after the operation. A **Contraction** operation, noted with “−”, eliminates a belief α from the epistemic state and those beliefs that make possible its deduction or inference. The sentences to eliminate might represent the minimal change on the epistemic state. Finally, a **Revision** operation (“∗”) inserts a consistent sentence to the epistemic state, guaranteeing a consistent outcome. This means that a revision adds a new belief and perhaps it eliminates other ones in order to avoid inconsistencies.

Other non-classical operations, like **Merge** [13] (“⊙”) fusions belief bases or sets assuring a consistent resulting epistemic state, and **Consolidation** (“Δ”) that restores consistency to a contradictory epistemic state. Usually, slight extensions or modifications of these operations are needed in order to capture different improved features of the environment required to work in. This is the case of the operator “φ” used to represent a **Kernel Revision by a Set of Sentences** operation, which defines a non-prioritized version of a kernel revision operation.

#### 3.1 Kernel Contractions

The **Kernel Contraction** operator, applicable to belief bases and belief sets, consists of a contraction operator capable of the selection and elimination of those beliefs in $K$ that contribute to infer $α$.

**Definition 3 (Set of Kernels [9])** Let $K'$ be a set of sentences and $α$ a sentence. The set $K'^{\perp}α$, called set of kernels is the set of sets $K'$ such that (1) $K' \subseteq K'$, (2) $K' \vdash α$, and (3) if $K'' \subset K'$ then $K'' \not\vdash α$. The set $K'^{\perp}α$ is also called set of $α$-kernels and each one of its elements are called $α$-kernel.
For the success of a contraction operation we need to eliminate, at least, one element of each \( \alpha \)-kernel. The elements to be eliminated are selected by an Incision Function.

**Definition 4 (Incision Function [9])** Let \( K \) be a set of sentences and “\( \sigma \)” be an incision function for it such that for any sentence \( \alpha \) it verifies, (1) \( \sigma(K^{+}\alpha) \subseteq \bigcup(K^{\pm}\alpha) \) and (2) If \( K' \in K^{+}\alpha \) and \( K' \neq \emptyset \) then \( K' \cap \sigma(K^{+}\alpha) \neq \emptyset \).

Once the incision function was applied we must eliminate from \( K \) those sentences that the incision has selected, i.e., the new belief base would consist of all those sentences that stayed outside of the scope of \( \sigma \).

**Definition 5 (Kernel Contraction [9])**

Let \( K \) be a set of sentences, \( \alpha \) a sentence, and \( K^{-}\alpha \) the set of \( \alpha \)-kernels of \( K \). Let “\( \sigma \)” be an incision function for \( K \). The operator “\( -\sigma \)” called kernel contraction determined by “\( \sigma \)” is defined as, \( K - \sigma \alpha = K \setminus \sigma(K^{-}\alpha) \).

Finally, an operator “\( -\)” is a kernel contraction operator for \( K \) if and only if there exists an incision function “\( \sigma \)” such that \( K - \alpha = K - \sigma \alpha \) for all sentence \( \alpha \).

### 3.2 Consistent Incorporation of Beliefs

A revision operator “\( * \)” looks for the addition of the new belief \( \alpha \) to the belief set \( K \), and therefore the assurance that the resulting belief set \( K * \alpha \) is consistent (unless \( \alpha \) is inconsistent). The first task can be accomplished by expanding by \( \alpha \), while for the second, by contracting by its complement “\( \sim \alpha \)”. If a belief set does not imply “\( \sim \alpha \)”, then \( \alpha \) can be added to it without loss of consistency. This composition of sub-operations gives rise to the following definition of a revision operator [7, 11]:

\[
(\text{Levi Identity}) \quad K * \alpha = (K - \sim \alpha) + \alpha
\]

We will define the revision operation in a set \( K \) regarding a sentence \( \alpha \), by means of the Levi Identity, assuming that “\( - \)” is a kernel contraction operator determined by an incision function “\( \sigma \)”.

**Definition 6 (Internal Kernel Revision [10])** Let “\( - \)” be a kernel contraction for a set \( K \). Then the Internal Kernel Revision operator for \( K \) is defined as \( K \mp \alpha = (K - \sim \alpha) + \alpha \).

Analogously, an External Kernel Revision operator is defined as \( K \pm \alpha = (K + \alpha) - \sim \alpha \). Finally, a kernel revision operator “\( * \)” may be characterized by either an internal “\( \mp \alpha \)” or an external “\( \pm \alpha \)” kernel revision.

### 3.3 Non-Prioritized Revisions

The classic revision operation is characterized by the postulates of rationality introduced by Grdenfors [8], some of them have been argued due to their arbitrariness. Particularly, the success postulate (\( \alpha \in K * \alpha \)) establishes that a new information to be revised in an epistemic state must be part of it, despite that other beliefs in the agent’s state must be eliminated in order to maintain its consistency. For that purpose is interesting to define new types of revision operations to “catch” the information in a more intuitively way such that a new information has “no absolute priority” over those in the epistemic state.

**Definition 7 (Explanation Set [5])** The set \( A \) is an explanation for \( \alpha \) iff it means a minimal proof for \( \alpha \), it is consistent and it is not self-explanatory, i.e., \( \alpha \not\in A \).

Usually one does not totally accept what others inform but only what one considers to be relevant. This property is known as partial acceptance, and its behavior may be modeled by a multiple revision operator as follows:

**Definition 8 (Non-Prior. Multi-Revision [5])** Let “\( \sigma \)” be an incision function, and \( K \) and \( A \), two consistent sets of sentences. The Non-Prioritized Kernel Revision by a Set of Sentences operator “\( \diamond_N \)” is defined as follows:

\[
K \diamond_N A = (K \cup A) \setminus \sigma((K \cup A) \pm \bot)
\]
4 Argument Revision Operators

Intuitively, a “Warrant-Prioritized Argument Revision Operator” (for short: WP Argument Revision Operator) revises a given program \( P = (\Pi, \Delta) \) by an external argument \( \langle A, \alpha \rangle \). Moreover, this argument ends up being warranted from the program resulting from the revision, provided that \( A \cup \Pi \) has a defeasible derivation for \( \alpha \). This requirement is justified as follows: the set \( \Pi \) of strict rules and facts represents (in a way) the current state of the world. The external argument \( \langle A, \alpha \rangle \) provides a set of defeasible rules that jointly with the state of the world decides in favor of the conclusion \( \alpha \), i.e., it poses a reason to believe in it. Hence, this argument does not stand by itself, but in conjunction with the strict part of the program it is being added to, i.e., \( \alpha \) is defeasibly derived from \( A \cup \Pi \). Besides its capability of deriving \( \alpha \), the argument should be consistent wrt. \( \Pi \), and minimal wrt. these two requirements. Formally:

**Definition 9 (External Argument Structure)** Let \( (\Pi, \Delta) \) be a DeLP-program, \( \langle A, \alpha \rangle \) is an external argument structure for a literal \( \alpha \) from \( (\Pi, \Delta) \) iff \( \langle A, \alpha \rangle \) is an (non-external) argument structure from \( (\Pi, \Delta \cup A) \).

If an argument that is non-compliant to this definition is introduced, then it is not ensured that the added argument is actually an argument in \( (\Pi, \Delta \cup A) \). Considering this new program, the question is how to warrant the new argument. Although it would be interesting to revise a program by \( \langle A, \alpha \rangle \) only when \( \alpha \) is not already warranted (by another argument), it might be desirable to have \( A \) as an undefeated argument. In our approach, we take this last posture: the WP Argument Revision Operator will ensure \( A \) to be an undefeated argument. In this way, \( \alpha \) would be always warranted.

For this matter, a hypothetical dialectical tree rooted on \( \langle A, \alpha \rangle \) is built. The D-tree is deemed as “hypothetical” due to \( \langle A, \alpha \rangle \) not belonging to \( P \). Incisions over arguments in this tree are made in order to turn \( A \) into an undefeated argument. Selections (consequently, incisions) must agree with some minimal change principle. In this way, we take a principle that attempts to ensure minimal deletion of the DeLP-program rules.

Finally, in the examples given throughout the article, we abstract away the argument comparison criterion: we will just give DeLP-programs, pointing out which is the associated D-tree for the argument being added, and thereafter the analysis begins.

4.1 WP Argument Revision

A WP Argument Revision Operator “\( \star_P^\Psi \)” attempts to insert an external argument \( \langle A, \alpha \rangle \) into a program \( P \), in such a way that \( \alpha \) turns out to be warranted from \( P \star_P^\Psi \langle A, \alpha \rangle \). Revising a program \( P = (\Pi, \Delta) \) by an external argument \( \langle A, \alpha \rangle \) involves the generation of a hypothetical D-tree rooted in \( \langle A, \alpha \rangle \), namely \( T_{P'}(\langle A, \alpha \rangle) \), where \( P' = (\Pi, \Delta \cup A) \). Therefore, since we want \( \alpha \) to be warranted, those undefeated defeaters for \( \langle A, \alpha \rangle \) will be cut off in order to turn \( \langle A, \alpha \rangle \) into an undefeated argument.

**Definition 10 (Argument Selection Function)** Let \( T = T_{P'}(\langle A, \alpha \rangle) \) be a D-tree and \( \lambda_i \) an argumentation line rooted in \( \langle A, \alpha \rangle \), then \( \gamma_P^\Psi(\lambda_i) = B \) iff \( B \) is a defeater for \( \langle A, \alpha \rangle \) marked as undefeated in \( T \). From now on, the argument selected in the \( i^{th} \) argumentation line will be called \( \Psi_i \).

In general, selecting defeaters for the root argument ensures a minimal deletion of defeasible rules from the DeLP-program at issue. That is because the deletion of a root’s defeater eliminates a whole branch. Trying to achieve the same result by deleting rules from “lower” arguments would affect a greater amount of arguments, due to the possibility of branching.

**Definition 11 (Argument Incision Function)** Let \( \Psi_i \) be the “less relevant interference argument” determined by an argument selection function \( \gamma_P^\Psi \) in the argumentation line \( \lambda_i \). Then a function \( \sigma_P^\Psi \) is an argument incision function iff it verifies \( \emptyset \subset \sigma_P^\Psi(\Psi_i) \subseteq \Psi_i \).

**Example 5** Let \( P_5 = (\Pi_5, \Delta_5) \) be a program where:

\[
\Pi_5 = \{ t, z \} \quad \Delta_5 = \begin{cases} 
\sim a \prec y, y \prec x, \sim a \prec z, \\
\sim a \prec w, w \prec y, \sim a \prec t 
\end{cases}
\]
Let us consider program \( P_5 \). Since arguments are minimal, given an argument for root-preservation, however, must be the single rule it contains:

\[
\{ \sim a \}
\]

Now consider that \( P_5 \) is revised by the external argument \( \langle A, a \rangle = \langle \{ a \prec x, x \prec z \}, a \rangle \). Then, from \( P_5' = (\Pi_5, \Delta_5 \cup A) \), we can build these new arguments:

\[
\begin{align*}
\langle B_1, \sim a \rangle & : \langle \{ a \prec y, y \prec x, x \prec z \}, \sim a \rangle \\
\langle B_2, \sim a \rangle & : \langle \{ a \prec w, w \prec y, y \prec x, x \prec z \}, a \rangle
\end{align*}
\]

From now on, we abstract away both the argument preference criterion and argumentation line acceptability, so we will just provide defeats between arguments with no further discussion: arguments \( B_1 \) and \( B_2 \) defeat \( A \), argument \( B_3 \) defeats \( B_2 \), and \( B_4 \) defeats \( B_3 \). Then, assume that the hypothetical D-tree in Figure 2 is built from \( P_5' \). Here the argument selection function \( \gamma_P \) selects the arguments \( B_1 \) and \( B_2 \) to be cut off, while the argument incision function \( \sigma_P^w \) applied over them could be any subset.

\[
\sigma_P^w(\Psi) \cap B \neq \emptyset \text{ holds, then } \sigma_P^w(\Psi) \cap B \text{ is called a collateral incision over } B.
\]

The argument incision function should be applied to the portion of the argument that does not belong to the root argument, i.e., it should avoid any collateral incision over the root argument. The motivation of this property is that if a rule belonging to the root argument were to be cut off, this argument would no longer hold, turning impossible to warrant its conclusion. Therefore, the following principle models the argument incision function “\( \sigma_P^w \):”

\( \text{(Root-Preservation)} \quad \sigma_P^w(\Psi_i) \cap A = \emptyset \), where \( \gamma_P^w(\lambda_i) = \Psi_i \)

**Example 6** Let us consider program \( P_5' \) from Ex. 5. Here, the incision over argument \( B_1 \) could be any subset that does not contain the defeasible rule \( x \prec z \) (which belongs to the root argument \( A \)), that is, any subset of \( \{ a \prec y, y \prec x \} \). For instance, \( \sigma_P^w(B_1) = \{ a \prec y \} \). The incision over argument \( B_2 \), however, must be the single rule it contains: \( \sigma_P^w(B_2) = \{ a \prec z \} \). Therefore we have that \( \sigma_P^w \) satisfies root-preservation.

**Remark 1** Since arguments are minimal, given an argument \( \langle B, \beta \rangle \), it is clear that \( B \setminus \sigma_P^w(B) \) is not an argument for \( \beta \).

When a collateral incision arises, some side effects may occur compromising the objective of the revision (i.e., the root argument might end up defeated). This may happen in case a collateral incision affects a supporting argument in an argumentation line which originally had a defeated defeater (for the root), thus yielding it undefeated.

However, collateral incisions are not always harmful; they could be utilized to achieve a positive effect. For instance, the deletion of one rule from an undefeated root’s defeater could also make another undefeated defeater disappear, which results in a better preservation of the program rules.
Remark 2 The marking of a D-tree is considered dynamic, this is, it may change by a collateral effect of the applied incisions. Thereafter, if the status of an argumentation line has changed (now having an undefeated defeater for the root), then it should be further affected by an incision function.

Definition 13 (Root-Preserving Incision) An argument incision function "\( \sigma^w_P \)" determined by an argument selection function "\( \gamma^w_P \)" is called root-preserving argument incision function if it verifies root-preservation.

Now that both the selection and the incision function are defined, the WP Argument Revision operation can be formally defined.

Definition 14 (WP Argument Revision) Let \( P \) be a program such that \( P = (\Pi, \Delta) \). A warrant-prioritized revision operation of \( P \) by an external argument \( (A, \alpha) \), namely \( P \star^\omega_P (A, \alpha) \), is defined by means of a root-preserving argument incision function "\( \sigma^\omega_P \)" as follows:

\[
P \star^\omega_P (A, \alpha) = (\Pi, A \cup \Delta \setminus \bigcup_i (\sigma^\omega_P(\Psi_i)))
\]

Theorem 1 Let \( P_R = P \star^\omega_P (A, \alpha) \) be a revised defeasible logic program by an argument \( (A, \alpha) \), then \( \alpha \) is warranted from \( P_R \).

Proof sketch: For each undefeated defeater in the hypothetical tree rooted in \( (A, \alpha) \) returned by the selection, the incision there makes that argument disappear, leaving the root argument either alone or being attacked by defeated defeaters. Therefore, the marking procedure indicates that the tree \( T_{P_R}(A, \alpha) \) warrants \( \alpha \).

Example 7 From Ex. 6 we have the incisions \( \sigma^\omega_P(B_1) = \{\sim a \prec y\} \) and \( \sigma^\omega_P(B_2) = \{\sim a \prec z\} \). Then, from the revision \( P_5 \star^\omega_P \{(a \prec x, x \prec z)\} \) made in Ex. 5 we have:

\[
P_{5R} = (\Pi_5, \Delta_5 \cup \{a \prec x, x \prec z\} \setminus \{a \prec y, \sim a \prec z\}) =
\]

\[
\left\{ \begin{array}{c}
(t, ) \cup \{ a \prec x, y \prec x, x \prec z, \\
( z, ) \cup \{ a \prec w, w \prec y, \sim a \prec t \}
\end{array} \right. 
\]

From \( P_{5R} \) literal \( a \) is warranted, since the D-tree is just the root of the one depicted in Ex. 5.

4.2 WP Argument Revision Considering Extended Arguments

Revising a program \((\Pi, \Delta)\) by an argument \( (A, \alpha) \) assuming that \( A \cup \Pi \) derives \( \alpha \) might be too restrictive. Then, from the operator explained in the last section, we can consider a variation of it that revises a program by an extended argument. These arguments will contain strict rules and facts, besides defeasible rules. This characteristic gives them the possibility of being self-contained, in the sense that they derive a conclusion just by themselves. However, extended arguments bring about a main drawback: consistency checking, i.e., when a program is revised by an extended argument, the join of their sets of strict rules must be non-contradictory. Thereafter, this join can be defined following several policies, i.e., deleting rules in contradiction, turn them into defeasible rules, etc. Moreover, this policy can be applied either over the strict rules of the program, over the strict rules of the argument, or both. Since we are defining a prioritized argument revision operator, we are going to keep the first option: only strict rules of the program will be affected. Note that the notion of external argument is no longer needed; the external argument will be inserted in such a way that it will always hold as an argument. Next, we formally define the notion of extended argument.

Definition 15 (Extended Argument) Given a set \( \Pi \) of strict rules and a set \( \Delta \) of defeasible rules, a pair \((\Pi, \Delta), \alpha\) is an extended argument structure for a literal \( \alpha \), if there is a minimal defeasible derivation for \( \alpha \) from \( \Pi \cup \Delta \), and \( \Pi \cup \Delta \) is non-contradictory.

An extended-argument revision operation is noted as \( P \star^\omega_P ((\Pi', \Delta'), \alpha) \), where \( P = (\Pi, \Delta) \). Then, we should consistently join both strict sets \( \Pi \) and \( \Pi' \), by means of a prioritized multiple revision operator, namely "\( \circ_P \)" , consequently defined as:
Given a K

Revising a program

Consider the extended argument

Remark 3

Example 8

Definition 17 (WP Extended-Argument Revision)

Proposition 1

Given a multiple prioritized revision operation K ⊢ A, the following properties hold:

1. A ⊆ K ⊢ A
2. K ⊭ K ⊢ A iff K ∪ A ⊢ ⊥
3. K ⊢ A \not\vdash ⊥

Therefore, as part of the definition of a Warrant-Prioritized Extended-Argument Revision Operation

Π ⊢ (Π′, ∆′, α) (where \( P = (Π, ∆) \)), we should achieve the consistent joint of both strict sets of rules Π and Π′, such that Π ⊢ Π′. Afterwards, the sets of defeasible rules ∆ and ∆′ should be joined. This may be easily achieved since there is no need to preserve consistence by a set of defeasible rules, then a preliminary version might be just ∆ ∪ ∆′. But furthermore, in order to preserve beliefs, a slight modification is proposed by adopting the policy of “weakening” the erased strict rules ρ, selected from Π by an incision function σ. Finally, while ρ ∈ σ(Π ∪ ∆) or equivalently ρ ∈ Π ∖ Π ′, the referred “weakening” is performed by means of a function δ such that δ(ρ) is the defeasible version of the strict rule ρ. This idea is originally exposed in [5], where a first approach of revision in argumentative systems is given, and also in [15], where at one stage of the architecture two DeLP- programs have to be combined.

Supposing that the operator “\( \circ _P \) ” would define a new program \( P_\circ = (Π \circ Π′, ∆ ∪ \delta(Π \setminus Π ′) ∪ ∆′) \), it could not be possible to ensure that α is warranted from \( P_\circ \). Therefore, in order to achieve warrant for α, we propose to define the operator “\( \circ _P \) ” by means of the operator “\( \circ _P \) ” previously defined:

Definition 17 (WP Extended-Argument Revision) A warrant-prioritized extended-argument revision operator “\( \circ _P \) ” is defined in terms of the operator “\( \circ _P \) ” and the WP Argument Revision operator “\( \circ _P \) ” as follows:

\( (Π, ∆) \circ _P (Π′, ∆′, α) = (Π ∪ ∆, ∆ ∪ \delta(Π ∖ Π ′)) \circ _P (Δ′, α) \)

Example 8 Consider the extended argument A = (\( \{x ← t, t\}, \{a ← x\} \), a) and the DeLP-program P8 = (\( \{\sim x ← w, w ← z, z, y\}, \{\sim a ← y\} \)). When joining A with P8, we have that there are strict derivations for both x (from the strict part of A) and \( \sim x \) (from the strict part of P8). Therefore, we apply the function δ to at least one strict rule from P8 that is involved in the derivation of \( \sim x \); for instance, \( δ(w ← z) = w ← z \). Now we have:

\[ P_8 \circ _P A = \left( \begin{array}{c}
\sim x ← w, \\
\sim a ← y, \\
\sim a ← y, \\
a ← x
\end{array} \right) \circ _P \left( \begin{array}{c}
\sim x ← w, \\
\sim a ← y, \\
\sim a ← y, \\
a ← x
\end{array} \right) \]

From this DeLP-program we can build the following extended arguments and dialectical tree:

\[ \{A, a\} : (\{x ← t, t\}, \{a ← x\}, a) \]
\[ \{B, \sim a\} : (\{y\}, \{\sim a ← y\}, \sim a) \]
\[ \{C, \sim x\} : (\{\sim x ← w, z\}, \{w ← z\}, \sim x) \]

Finally, as explained in the previous section, incisions over arguments B and C have to be made in order to turn A into an undefeated argument.

Remark 3 Revising a program \( P \) by an extended argument \( (Π′, ∅, α) \) is the case of an argument that, once introduced into \( P \), would have no argument against it (i.e., by definition, there would be no arguments for \( \sim α \)), since α would have a strict derivation from Π′. Therefore, strict derivations for \( \sim α \) are to be weakened into defeasible derivations that, although they have no effect as arguments for \( \sim α \), they can be a part of other derivations that should not be “broken”. This stresses the importance of not deleting conflicting strict rules, which can have undesirable collateral effects.
5 Discussion

The different versions of the argument revision operator for defeasible logic programs here proposed are mostly based on others from the classical theory change. While selections may be related to the partial-meet contractions theory [2], incisions are inspired by kernels contractions [9]. Furthermore, the order established by a preference criterion on selections may be possibly related to safe contractions originally exposed in [3], and later on related to kernel contractions in [10]. Indeed, an argument is a kind of kernel or minimal proof for a given consequence. These concepts are more deeply treated in [4], where the Kernel Revision by a Set of Sentences is proposed. Moreover, this operator constitutes part of the inspiration for the argument revision operators here exposed.

Our definition of the "α" operator ensures that there is no inconsistent intermediate epistemic state during the revision process. In [5], a non-prioritized revision operator over explanations "α" is introduced, which does generate an inconsistent intermediate epistemic state when the revision is performed. The latter operator justifies the non-prioritization by the assertion that there is no reason to accept new information blindly, discarding older beliefs without proper justification. We agree with this posture, but since we are defining an operator that has the objective of warranting the conclusion of the newly inserted argument, we have to give priority to newer information. Suppose we define a non-prioritized version of our operator by means of an operator "α":

\[(\Pi, \Delta) \ast_{\alpha} ((\Pi', \Delta'), \alpha) = (\Pi \circ_{\alpha} \Pi', \Delta \cup \delta((\Pi \cup \Pi') \setminus (\Pi \circ_{\alpha} \Pi'))) \ast_{\alpha} (\Delta', \alpha)\]

Note that the matter of de-prioritizing the incorporation of new information seems to be attained just to the join of the strict rules sets, while when we refer it as "warrant-prioritized", a reference to the priority of giving warrant to the new conclusion \(\alpha\) is given. However, the definition of the non-prioritized operator has a major flaw: it does not ensure the warrant of \(\alpha\). An example will clarify this assertion:

Example 9 Consider the extended argument \(\mathcal{A} = \langle (\Pi_9', \{\}), a = \{\{a \leftarrow b, b\}, \{\}\} \rangle\) and the DeLP-program \(\mathcal{P}_9 = (\Pi_9, \{\}) = \{\{\sim a \leftarrow c, c\}, \{\}\}\). Here, if we prefer some older information over new one, we will have no argument for "\(\alpha\)" from \((\Pi_9 \circ_{\alpha} \Pi_9', \delta((\Pi_9 \cup \Pi_9') \setminus (\Pi_9 \circ_{\alpha} \Pi_9')))\), no matter if the rule \(a \leftarrow b\) is turned into a defeasible rule (see Remark 3). Not having an argument for "\(\alpha\)" makes impossible to have a warrant for it.

6 Conclusions & Future Work

In this work we have presented two different approaches for the WP Argument Revision operator: one considering regular arguments, and another, considering extended arguments. Both operators have a common theoretical basis, but the latter one has to resolve some extra issues.

In general, the theory we are defining cannot be trivially related to the basic concepts of belief revision. Regarding the basic postulates for a revision operator, as originally exposed in [2], a deep analysis is required. For example, the success postulate \((K * \alpha \vdash \alpha)\) makes reference to a knowledge base \(K\), which in our case is a DeLP-program \(\mathcal{P} = (\Pi, \Delta)\). For both of the argument revision operators here proposed, success is defined analogously, where the consequence notion is the warrant of the conclusion of the argument being added to \(\mathcal{P}\). This statement is verified by Theorem 1. Another interesting postulate to be analyzed is consistency, which states that the outcome of a revision \(K * \alpha\) must be consistent if \(\alpha\) is non-contradictory. In our proposal, this postulate is treated in a trivial manner, since programs are revised by arguments, which are consistent by definition. Regarding extended arguments, a join between the strict parts of the program and the argument is performed to ensure consistency. DeLP-programs are divided in two subsets of rules: \(\Pi\) and \(\Delta\), where only \(\Pi\) is required to be consistent, which is not modified by any of the argument revision operators we propose (because arguments do not introduce strict rules). Finally, the consistency postulate always hold for the two argument revision operators.

Besides exposing a complete list of the basic postulates and the respective axiomatic representations for each argument revision operator, future work also includes the definition of contraction/expansion of a DeLP-program by an argument, and a detailed study towards the possibility of duality between the operators of contraction/expansion and revision in argumentation systems.
Optimality is not a property pursued in this work, since there are some cases in which incisions made in a lower level might compromise a smaller amount of program rules. Our major concern here is to define revision operators that are correct regarding the objectives exposed above. That is, the main objective of this article is to present a first approach for revising defeasible logic programs by an argument. Other possibilities besides warrant-prioritized argument revision are left unaddressed in this paper, as well as variations of the operators here defined. Some of these options are interesting, whereas others are trivial; however, the whole range of possibilities cannot be accounted on a single article. For instance, regarding the minimal change principle, at least two options arise: (1) we might introduce the notion of epistemic importance in an argument level, and thus would incise first those arguments that are less important wrt. the total epistemic order among them; (2) provided that D-trees are an important tool to understand the interrelation among arguments and their influence to the final answer, we might want to preserve the structure of the hypothetical D-tree; hence, incisions would be performed in its lowest levels.

References